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Thermal Analysis of the Heat Exchangers and Regenerator in Stirling Cycle Machines

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A closed-form expression for the effectiveness of the heat exchangers and regenerator of a Stirling cycle machine is given. This result may be used in a simple way to evaluate their effect on the machine performance. The proposed method allows the actual cycle gas temperatures in the heater and cooler to be obtained readily, once the geometry of the heater, cooler, and regenerator is known and some quantities characterizing the engine dynamics (strokes, frequency, and phase angle of the moving elements) and its heat-exchange processes (inlet temperatures of the heating and cooling fluids and their volumetric flow rates) are measured. Thus, an immediate indication about the effectiveness of the heat exchangers and regenerator as well as about the machine thermal efficiency may be obtained. The availability of a closed-form expression for the heater, regenerator, and cooler effectiveness is useful especially for those engines, like the free-piston Stirling engines, whose design requires the application of analytically based optimization criteria. The obtained relations have been applied to the well-known Space Power Research Engine.

Nomenclature

A = cross-sectional area for moving element
A_f = cross-sectional area for flow
A_R = relative amplitude of fluid displacement, $y_{\max}/L$
C = heat capacity rate, $c_p\dot{m}$
c_p, c_v = specific heats at constant pressure and volume
$c_s\rho_s$ = heat capacity of solid per unit of volume
d_h = hydraulic diameter, $4A_fL/S$
f = operating frequency, Hz
h = convective heat transfer coefficient
k = thermal conductivity
L = heat exchanger length
$\dot{m}$ = mass flow rate, $\rho\dot{V}$
N_T = temperature ratio, $T_{w,h}/T_{w,k}$
N_{TC} = ratio of heat capacity of regenerator matrix to heat capacity of tidal gas
Nu = Nusselt number, hd_h/k
P = amplitude-average pressure ratio, $(p_{w,\max} - \bar{p}_w)/\bar{p}_w$
Pr = Prandtl number, $c_p\mu/k$
p = pressure
Q = heat exchanged
R = specific gas constant
Re = Reynolds number, $\dot{m}d_h/(\mu A_f)$
Re_ω = dimensionless frequency, $\rho\omega d_h^2/(4\mu)$
r = displacer-piston stroke ratio, X_d/X_p
S = heat transfer area
T = temperature
t = time
V = volume
$\dot{V}$ = volumetric flow rate
W = work exchanged
X = stroke
x = moving element displacement
y = streamwise coordinate
$y_{\max}$ = amplitude of fluid displacement in heat exchangers
β = heat capacity rate ratio, $C_{\min}/C_{\max}$
β_F = beta function
γ = ratio of specific heats, c_p/c_v
ε = effectiveness
η = efficiency
μ = dynamic viscosity
ξ = regenerator porosity, void volume/total volume
ρ = density
Φ = ratio of tidal mass of gas to mass of gas resident in regenerator
ϕ = piston-displacer phase angle
ω = angular frequency, $2\pi f$ rad/s

Subscripts
c = compression space
d = displacer
e = expansion space
h = heater
hf = heating fluid
h/rg = heater/regenerator interface
i = inside
in = inlet
k = cooler
kf = cooling fluid
max = maximum
mf = metal felt matrix
min = minimum
o = outside
out = outlet
p = power piston
r = displacer rod
rg = regenerator
ro-hi = regenerator outlet-heater inlet
w = working fluid
ws = woven screen matrix

Superscripts
$\dot{}$ = time rate of change
$\bar{}$ = average over a cycle
$\hat{}$ = average over half a cycle

Introduction

IN the Stirling cycle simulation analyses presented and published in the literature,[1–3] the heat transfer between the outside surface of the heat exchanger and the outside fluid has been fully neglected, with the exception of the analysis de-

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veloped in Ref. 4, where the effect of the various thermodynamic losses on the performance of a free-piston Stirling machine was estimated.

In this paper, the cycle gas temperatures of the heater and cooler have been linked to the volumetric flow rates and inlet temperatures of the outside (heating and cooling) fluids by means of the effectiveness-number of transfer units (NTU) method. In other words, they may be calculated as a function of the following quantities:

1) Heat exchanger geometry (tube length, number of tubes, inside and outside heat transfer surfaces).

2) Cyclic average value of the convective heat transfer coefficient of the working fluid at the considered heat exchanger.

3) Heat capacity rate of the outside fluid (heating or cooling fluid) and its heat exchange coefficient and inlet temperature.

4) Heat transfer rate exchanged at the heater or cooler, which may be obtained applying the energy balance equation to the considered heat exchanger (cycle fluid side).

The volumetric flow rates and the inlet temperatures of the outside (heating and cooling) fluids represent the actual operational parameters of the machine, together with the load. Therefore, the main difference of the proposed thermal analysis with respect to others[5–8] is that the wall temperatures of the heater and cooler have not been assumed to represent the operational parameters of the machine, that is, the external heat source and cold sink. In fact, the values of these wall temperatures change when the actual operating conditions of the engine change.

Thus, the analytical thermal procedure developed in this paper allows the heat flux between the outside surface of the heat exchanger and the outside fluid to be fully considered for a proper design of the heat exchangers: when designing, the dead space introduced by the heater and cooler in the working gas circuit must be kept to a minimum to avoid a low volumetric compression ratio. In fact, it was found that the optimum compression ratio for Stirling engines ($\Rightarrow$ optimum cycle efficiency) is between 1.5–2.5 for the more common working fluids.[9] Obviously, a minimum dead space precludes the use of large heat exchangers, but, at the same time, demands a high heat flux. Therefore, finned heat transfer surfaces are convenient in most Stirling heaters and coolers.

The basic method herein proposed, giving a closed-form expression for the effectiveness of the heat exchangers and regenerator, may be very useful for a ready estimation of the thermodynamic behavior of the Stirling machine, especially for those engines (free-piston engines) whose design requires an analytical description. Numerical simulations based on time-stepping integration techniques of the dynamic and thermodynamic behavior of free-piston Stirling engines (FPSEs) can lead to accurate results for the performance prediction of a working engine, but cannot be used for design purposes because they do not give any indication of the choice of the various parameters involved in the machine behavior. In other words, they do not give any criterion for the cyclic steady operation of the engine.[10,11] From this point of view, the adoption of linearization techniques for the equations of motion of the moving elements of the machine (power piston and displacer) is much better because they allow one to analytically solve these equations and thus to find useful algebraic relations. These relations may be used not only for the performance prediction, but above all for the design of the engine to ensure it a good general performance and both a periodic steady and quite stable operation.[12–14]

The proposed approach implies the calculation of the various dimensionless parameters that affect the performance of the heater, cooler, and regenerator. This calculation is, in particular, carried out in a closed form on the basis of some simplifying assumptions, and taking account of the heat transfer correlations for oscillating-flow conditions in the Stirling heat exchangers and regenerator (working fluid side). The next step is to derive, always in an analytical form, the cycle gas temperatures in the heater and cooler, and hence, the thermal engine efficiency. The effect of each heat-exchange effectiveness on the machine performance may also be evaluated.

Finally, a test case is presented at the end of this paper with reference to the Space Power Research Engine (SPRE) FPSE.

Effectiveness in Heat Exchangers and Regenerators

As is well-known, in a heat exchanger the hot and cold fluids flow on either side of a thin partition, whose function is merely to separate the two fluids, and the heat is transferred from one to the other through this partition.

For steady operation, the performance of a heat exchanger is considered in terms of ε, which depends on two dimensionless parameters: NTU and β. The effectiveness charts of various flow arrangements and their functional relationships are shown in Ref. 15. In particular, when $C_{\max} \to \infty$ (that is, $\beta \to 0$), the ε-NTU expressions for the parallel flow, counterflow, and crossflow (with both fluids unmixed, or one fluid mixed and the other unmixed) cases simplify to

$$\varepsilon = 1 - e^{-\mathrm{NTU}} \tag{1}$$

Therefore, the flow arrangements listed before are not important where one heat capacity rate is very much greater than the other. The number of transfer units is defined as NTU = $U_o S_o / C_{\min}$, where U_o is the overall heat transfer coefficient based on the outside surface area S_o ($U_o S_o = U_i S_i$):

$$1/(U_o S_o) = 1/(h_i S_i) + 1/(h_o S_o)$$

In the previous expression the effects of the fouling and tube wall resistance to heat flow have been neglected for the sake of simplicity. If the outside and inside heat transfer surfaces are finned, the reader may refer to Ref. 16.

The convective heat transfer coefficients h_i and h_o are complex functions of the surface geometry, fluid properties (which are temperature-dependent), and flow conditions (e.g., either turbulent, transitional, or laminar flow regime). These coefficients are presented in graphical or functional form employing dimensionless parameters.[15,16]

In heat regenerators (fixed-bed or rotary), the hot and cold fluids pass alternatively over a solid wall: the wall absorbs heat when the hot fluid flows over it, and subsequently gives up this heat to cold fluid, the process being repeated cyclically. In practical applications, it is the final periodic steady state that is of interest. Hence, for cyclic equilibrium operation, which is reached when the heat transferred to the matrix during the flow of the hot gas stream is equal to the heat released from the matrix during the flow of the cold stream, the regenerator performance is considered in terms of effectiveness ε. It may be defined as the ratio of the heat actually exchanged to an ideal amount of heat that would be exchanged if the temperature of the cold gas could be increased to the entrance temperature of the warm gas.[17]

For the unidirectional operation of regenerators (fixed-bed or rotary), the effectiveness may be evaluated by means of the exact analytical solution given as a function of two dimensionless variables, NTU and $N_{\mathrm{TC}}\Phi$.[18] For countercurrent operation, instead, the effectiveness may be calculated as shown by Nusselt[19] and Jakob.[17] An approximate closed-form expression for the effectiveness of countercurrent fixed-bed regenerators used in Stirling machines was developed by Rea and Smith,[20] Qvale and Smith,[21] and Jones.[22,23] The analysis of Qvale and Smith[21] was modified by Jones[22,23] to take account of the temperature variation of the regenerator matrix. According to its theory, which considers the matrix having finite mass, the performance of a Stirling regenerator may be considered, in the author's opinion, as a function of the following dimensionless groups: NTU, N_{TC}, Φ, N_T, P, ϕ_p, γ, and n. They are defined as

$$\mathrm{NTU} = \frac{4k_w K \hat{R}e^n L A_f}{c_{p,w}\hat{\dot{m}}_w d_h^2} \tag{2}$$

$$N_{TC} = \frac{(1-\xi)c_s\rho_s L A_f}{\xi c_{p,w}\hat{\dot{m}}_w/(2f)} \tag{3}$$

$$\Phi = \frac{\hat{\dot{m}}_w/(2f)}{\bar{\rho}_w L A_f} \tag{4}$$

The quantities n and K are, respectively, the exponent and the coefficient in the regenerator heat transfer correlation:

$$Nu(t) = KRe(t)^n \tag{5}$$

The ϕ_p phase angle represents the phasing of the cycle gas pressure with respect to the mass flow rate passing through the regenerator. These quantities were assumed to vary sinusoidally according to the following equations[22]:

$$p_w(t) = \bar{p}_w - (p_{w,\max} - \bar{p}_w)\cos(\omega t - \phi_p) \tag{6}$$

$$\dot{m}_w(t) = \dot{m}_{w,\max}\sin\omega t \tag{7}$$

Finally, the regenerator ineffectiveness, $1-\varepsilon$, is given by[22,23]

$$1 - \varepsilon = (1 - \varepsilon_\infty) + \varepsilon_{HP} \tag{8}$$

where $(1-\varepsilon_\infty)$, the ineffectiveness of the matrix, if of infinite mass, is given by

$$1 - \varepsilon_\infty = \frac{1}{2\mathrm{NTU}}\left(\frac{\pi}{2}\right)^{1-n}\left[\frac{2(1/\gamma - 1)P}{\pi(\ell n\, N_T)\Phi}\cos\phi_p + \frac{N_{TC}}{N_{TC}-1}\right]\beta_F\left(\frac{3-n}{2},\frac{1}{2}\right) \tag{9}$$

and ε_{HP}, a correction term reflecting the effect of temperature swing in the regenerator matrix, is given by

$$\varepsilon_{HP} = \frac{\pi}{2}\left(1-\frac{1}{\gamma}\right)\frac{P}{(\ell n\, N_T)\Phi N_{TC}}\sin\phi_p \tag{10}$$

The regenerator effectiveness is independent of thermal diffusivity of the matrix. In fact, it was assumed that there was infinite conductivity of the solid perpendicular to the direction of flow and zero conductivity in that direction.[22]

However, following the mathematical procedure developed by Jones,[22] it was found that Eq. (9) is affected by an error that does not allow a right calculation of the matrix ineffectiveness $(1-\varepsilon_\infty)$. Its value may be estimated rightly by means of the following expression:

$$1 - \varepsilon_\infty = \frac{1}{\mathrm{NTU}}\left(\frac{\pi}{2}\right)^{1-n}\left[\frac{2(1/\gamma - 1)P}{(\ell n\, N_T)\Phi}\cos\phi_p + \frac{N_{TC}}{N_{TC}-1}\right]\beta_F\left(\frac{3-n}{2},\frac{1}{2}\right) \tag{11}$$

Therefore, when substituting Eq. (11) in Eq. (8), a lower value for the regenerator ε is obtained.

A complete description of Stirling regenerators has been recently proposed by Organ,[24,25] but closed-form expressions for the regenerator matrix effectiveness are not available.

Heat Exchangers and Regenerators in Stirling Machines

In Stirling machines the working gas flow inside the heater and cooler is unsteady, since it moves in an oscillating way between the expansion and compression spaces. Instead, the heating fluid flow outside the heater and the cooling fluid flow outside the cooler are steady.

The heaters used in fuel-powered Stirling machines are multitubular heat exchangers, where the bank of tubes is generally reduced to a single row positioned around the periphery of the cylinder, as shown in Fig. 1 (1-98 Philips engine[26]). The spacing between the heater tubes is much smaller than commonly used in heat exchangers with tube bundles. The flame gases pass over the tubes, whereas the working gas passes through the tubes and is carried back and forth between the hot end of the regenerator and the expansion space. The flame gases flow over the tubes more or less parallel to the cycle gas passage.

Other Stirling engines use several heater units positioned around the periphery of the cylinder (GPU-3 General Motors engine), and a crossflow arrangement is realized.

A heater heat exchanger, very simple in shape and easy to make, and a burner system appropriate to this heater are shown in Ref. 27. The heat input system includes a heater shell shaped like a U-cup and a flame tube located in the heater shell. The heat transfer correlations are presented in a functional form employing dimensionless groups.

The heater configuration usually used in solar-energy-powered Stirling machines is a large array of staggered and closely packed tubes located between tube sheets in an annular region around the displacer assembly (Fig. 2). The working gas is on the inside of the tubes and the molten salt heating fluid circulates circumferentially over the outside surfaces of the tubes,

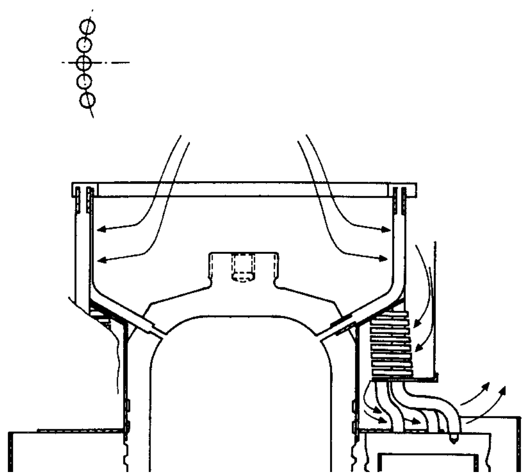

Fig. 1 Heater with a single row of tubes: parallel flow arrangement.[26]

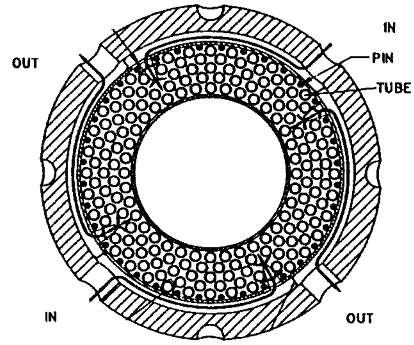

Fig. 2 Annular heat exchanger with a large array of staggered tubes: crossflow arrangement.[28]

passing more or less transversely between them. Thus, a crossflow arrangement is realized.

The coolers in most Stirling engines take a fairly traditional form: a bank of tubes carries the working gas to be cooled while the cooling fluid passes between them.

Many Stirling engines use several cooler units positioned around the periphery of the cylinder. In this case the coolant passes more or less transversely between the tubes of each cooler unit. In this case each cooler element is joined in tandem to the regenerator (regenerator–cooler unit), whereas the heater can be single or equal to the number of coolers. As an example, the 1-98 Philips engine has six regenerator–cooler units and a single heater. The GPU-3 General Motors engine has, on the contrary, eight cooler–regenerator–heater units for each cylinder.

In other Stirling engines, a single annular cooler (and regenerator) around the cylinder is used, as shown in Fig. 2. In this case the cooler pipes are narrow and uniformly spaced, with sufficient spacing to allow the passage of the cooling fluid. The coolant passes either transversely to the tubes of the cooler unit, as shown in Fig. 2 (SPRE engine), or parallel to them (Sunpower RE-1000 engine).

The regenerator matrices commonly used in Stirling machines are fixed-bed. There are two types: 1) simple, rectangular woven-screen matrices and 2) metal–felt matrices. However, in small Stirling machines like the Thermo-mechanical Generator of Cooke-Yarborough,[29] the clearance between the displacer and the cylinder wall works as a regenerator.

Heat Transfer Correlations in Stirling Machines

To calculate the effectiveness of the heat exchangers and regenerator, the following assumptions have been made:

1) The working fluid temperatures in the heater $T_{w,h}$ and cooler $T_{w,k}$ are spatially uniform and time independent, as shown in Fig. 3.[5]

2) The temperature profile through the regenerator is linear, as shown in Fig. 3.[5] Therefore, the mean effective cycle gas temperature in the regenerator $T_{w,rg}$, can be expressed as a logarithmic mean of $T_{w,h}$ and $T_{w,k}$.

3) The ideal gas equation applies (c_p and c_v are independent of temperature).

4) The cycle gas density in the heat exchangers and regenerator is constant.

5) The cycle gas pressure is linearized.

6) A beta-type machine (displacer and power piston in the same cylinder) is considered.

7) A cyclic steady state is established, characterized by sinusoidal laws of motion of the piston and displacer

$$x_d(t) = (X_p/2)r \sin \omega t \qquad x_p(t) = (X_p/2)\sin(\omega t - \phi) \quad (12)$$

Starting from the first assumption, the β capacity rate ratio is always equal to zero, independently of the type of heat exchanger used in the machine. Therefore, Eq. (1) is valid, where $C_{\min} = C_{hf}$ in the heater and $C_{\min} = C_{kf}$ in the cooler.

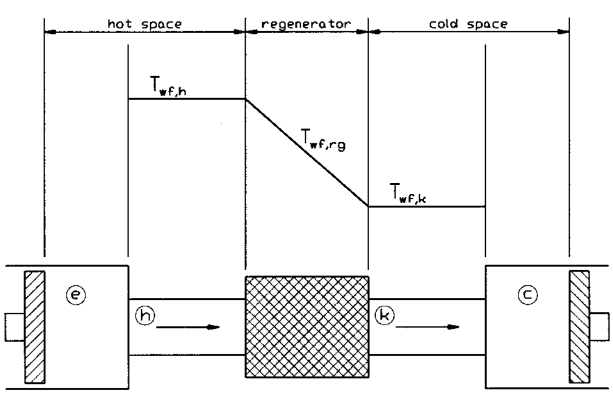

Fig. 3 Temperature profile of the working fluid in the heat exchangers and regenerator of a Stirling machine.

The problem consists of the evaluation of both the inside and outside heat transfer coefficients for the heater and cooler, and of the heat transfer coefficient for the regenerator. This evaluation in particular requires the knowledge of the Reynolds number for the working gas oscillating flow. Bearing in mind the Reynolds number definition (see the Nomenclature), $\dot{m}_w$ through the heat exchangers and regenerator may be calculated, in case of sinusoidal motions of the piston and displacer and considering positive the flow from the expansion space to the compression one (Fig. 3), as indicated by Benvenuto and de Monte[30]:

$$\dot{m}_w(t) = \dot{m}_{w,\max} \sin(\omega t - \phi_m) \quad (13)$$

where $\dot{m}_{w,\max}$ is the amplitude of $\dot{m}_w(t)$, and ϕ_m is its phase-angle with respect to the displacer motion:

$$\dot{m}_{w,\max} = \bar{\rho}_w(X_p\omega/4)[A_p^2 - 2A_p(2A_d - A_r)r \cos \phi + (2A_d - A_r)^2 r^2]^{1/2} \quad (14)$$

$$\tan \phi_m = r(2A_d - A_r)(A_p \sin \phi)^{-1} - \cot \phi \quad (15)$$

Applying the ideal gas law:

$$\bar{\rho}_w = \bar{p}_w/(RT_w) \quad (16)$$

where $T_w = T_{w,h}$ in the heater, $T_w = T_{w,k}$ in the cooler, and $T_w = T_{w,rg}$ in the regenerator. Therefore, the amplitude of the mass flow rate depends on the considered heat exchanger by means of $\bar{\rho}_w$. Similarly, the amplitude of the instantaneous Reynolds number

$$Re(t) = Re_{\max} \sin(\omega t - \phi_m) \quad (17)$$

where $Re_{\max} = d_h\dot{m}_{w,\max}/(\mu_w A_f)$. In addition, it is easy to verify that

$$\hat{\dot{m}}_w = (2/\pi)\dot{m}_{w,\max} \qquad \hat{Re} = (2/\pi)Re_{\max} \quad (18)$$

On the basis of last assumption, it may be said that the method is particularly suitable for those engines that are characterized by sinusoidal laws of motion of the moving elements, like the FPSEs.[29]

Inside Heat Transfer Coefficient

Recently, considerable attention has been given to the study of oscillatory heat transfer in a periodically reversing pipe flow with applications to Stirling machines. A correlation equation for the cycle-averaged Nusselt number in an oscillating and reversing flow has been proposed by Tang and Cheng.[31] As far as Tang and Cheng[31] are aware, this is the first correlation equation taking account simultaneously of the following three similarity parameters: $Re_{\max}$, Re_ω, and A_R. The correlation obtained employing a multivariate statistical method is

$$\overline{Nu} = -0.494 + 0.0777[A_R/(1 + A_R)]^2\hat{Re}^{0.7} - 0.00162\hat{Re}^{0.4}(4Re_\omega)^{0.8} \quad (19)$$

where $\hat{Re}$ is given by Eq. (18), and A_R, for sinusoidal fluid motion, by[32]

$$A_R = \tfrac{1}{4}(d_h/L)(Re_{\max}/Re_\omega)$$

The empirical correlation (19) is obviously valid under the range of tested conditions[31]: $Pr = 0.7$, $d_h/L = 0.0185$, $11 < Re_{\max} < 10{,}995$, $1.75 < Re_\omega < 45$, and $0.03 < A_R < 1.13$. The Prandtl number for air, helium, and hydrogen, having application to Stirling working gas circuit, is fairly constant with the temperature, and equal to 0.7. Note that at steady state where $y_{max} \to \infty$ (and, therefore, $A_R \to \infty$) and $Re_\omega = 0$, Eq. (19) reduces to a correlation equation similar in form to those of a steady turbulent flow.

The thermal properties of the working gas (density, dynamic viscosity, and thermal conductivity), which appear in the dimensionless groups of the heat transfer correlation, have to be calculated at $T_{w,h}$ for the heater and $T_{w,k}$ for the cooler. The working fluid properties as a function of temperature are given in Ref. 33.

Outside Heat Transfer Coefficient

The outside heat transfer correlation for both in-line and staggered tube arrangements and for tube bundles having 10 or more transverse rows ($N \geq 10$) in the direction of flow has an expression of the following form[16]:

$$Nu = 1.13C_oRe^nPr^{1/3} \tag{20}$$

for $2 \times 10^3 < Re < 4 \times 10^4$, $Pr > 0.7$. Here, $Nu = hD/k$ and $Re = \rho Du_{max}/\mu$, where D is the o.d. of the tube, and u_{max} is the maximum flow velocity based on the minimum free-flow area formed by adjacent heater tubes. The values of constant C_o and the exponent n, which appear in correlation (20), are presented in a tabular form in Ref. 16. In case of noncircular tubes of the bank, the o.d. D has to be replaced by an equivalent diameter D_e, which depends on the cross-sectional geometry of the tube. For tube bundles having $N = 1$, transverse rows in the direction of flow, Eq. (20) becomes

$$Nu = 0.64(1.13C_oRe^nPr^{1/3}) \tag{21}$$

In the heater of a fuel-powered Stirling engine, having a single row of closely spaced tubes (Fig. 1), the outside heat transfer coefficient was always found to be several times more than predicted on the basis of the empirical formula (21). This discrepancy has been primarily attributed to 1) nonuniform flow of the flame gases between the tubes of the heater cage (because of the very close spacing), 2) radiation from the flame itself and by the hot walls of the combustion chamber, and 3) flow of flame gases more or less parallel to the tubes of the heater. For the heater cage illustrated in Fig. 1, and for a mean flame temperature of about 1800°C (1-98 Philips engine), the data yield a straight line for $300 < Re < 1.6 \times 10^3$, which is represented by[26]

$$Nu = 0.24Re^{0.75} \tag{22}$$

Using this expression, there is much better agreement between the predicted and measured heat transfer. Thus, correlation (22) is a good rule for situations similar to Fig. 1.

The outside heat convective coefficient in the cooler tubes of a Stirling machine may be predicted on the basis of the empirical formula (20), when the coolant passes transversely between the tubes. On the contrary, when it passes parallel to the tubes of the cooler, the Dittus–Boelter correlation may be applied: $Nu = 0.023Re^{0.8}Pr^{0.4}$, valid for fully developed turbulent flow inside smooth ducts, and for $0.7 < Pr < 160$. Since the cooler cross section for the cooling flow is not circular, as shown in Fig. 2, the turbulent flow may occur for $Re > 2.3 \times 10^3$, where the Reynolds number is based on the hydraulic diameter, as well as the Nusselt number.

Once $\bar{h}_i$ and h_o are determined for the considered heat exchanger (heater or cooler), it is possible to calculate the NTU and, therefore, the effectiveness by Eq. (1).

Regenerator Heat Transfer

To evaluate the Stirling regenerator effectiveness given by Eq. (8), with positions (10) and (11), the following quantities have to be calculated: $(p_{w,max} - \bar{p}_w)$, $\bar{p}_w$, $\bar{\rho}_w$, ϕ_p, n, and K. They appear in the regenerator effectiveness expression both explicitly as ϕ_p, n, and implicitly as $(p_{w,max} - \bar{p}_w)$, $\bar{p}_w$, $\bar{\rho}_w$, n, and K, by means of the dimensionless parameters that affect the regenerator performance.

Linearizing the cycle gas pressure[34] and assuming sinusoidal the motions of the piston and displacer [see Eq. (12)], the following relation is obtained:

$$p_w(t) = \bar{p}_w - (p_{w,max} - \bar{p}_w)\cos(\omega t - \phi_{pd}) \tag{23}$$

where $(p_{w,max} - \bar{p}_w)$ is the amplitude of $p_w(t)$, and ϕ_{pd} is its phase angle with respect to the displacer motion:

$$p_{w,max} - \bar{p}_w = \left[\left(\frac{\partial p_w}{\partial x_p}\right)_{0,0}^2 + 2\left(\frac{\partial p_w}{\partial x_p}\right)_{0,0}\left(\frac{\partial p_w}{\partial x_d}\right)_{0,0} \times r\cos\phi + \left(\frac{\partial p_w}{\partial x_d}\right)_{0,0}^2 r^2\right]^{1/2}\frac{X_p}{2} \tag{24}$$

$$\tan\phi_{pd} = -r\left(\frac{\partial p_w}{\partial x_d}\right)_{0,0}\left[\left(\frac{\partial p_w}{\partial x_p}\right)_{0,0}\sin\phi\right]^{-1} - \cot\phi \tag{25}$$

The average pressure $\bar{p}_w$ and the first-order partial derivatives of the working gas pressure calculated in $(x_p, x_d) = (0, 0)$ depend on the thermodynamic model (e.g., either isothermal or adiabatic) selected for the working gas circuit. They may be evaluated as shown in Refs. 34 and 35. The density $\bar{\rho}_w$, appearing in Eq. (4), is given by Eq. (16) simply setting $T_w = T_{w,rg}$. The pressure variation may be written also in the form:

$$p_w(t) = \bar{p}_w - (p_{w,max} - \bar{p}_w)\cos[(\omega t - \phi_m) - \phi_p] \tag{26}$$

where $\phi_p = \phi_{pd} - \phi_m$ is the phasing of the cycle gas pressure with respect to the mass flow rate given by Eq. (13). The phase-angles ϕ_m and ϕ_{pd} are given, respectively, by Eqs. (15) and (25).

Heat transfer correlations for oscillating-flow conditions in Stirling regenerators have been recently presented by Gedeon and Wood.[36] They found that instantaneous local Reynolds numbers appear to characterize quite adequately the heat transfer for oscillating-flow conditions in Stirling regenerators. The exception to this might be for dimensionless frequencies above about 20, but in most Stirling regenerator applications Re_ω is less than this value. Therefore, neglecting enhanced axial conduction, the oscillating-flow Nusselt numbers for woven-screen and metal-felt are, respectively,[36]

$$Nu_{ws}(t) = [1 + 0.64Pr^{0.72}Re(t)^{0.72}]\xi^{1.79} \tag{27}$$

$$Nu_{mf}(t) = [1 + 0.48Pr^{0.79}Re(t)^{0.79}]\xi^{2.75} \tag{28}$$

The listed empirical correlations are obviously valid under the range of tested conditions,[36] which are, however, representative of most Stirling machines: $1.04 < Re_{max} < 3.4 \times 10^3$, $0.0048 < Re_\omega < 16$, $0.17 < A_R < 3.0$ for woven screens, and $0.79 < Re_{max} < 1.4 \times 10^3$, $0.0037 < Re_\omega < 3.3$, $0.17 < A_R < 3.8$ for metal felts.

The constant 1 was introduced in Nusselt number expressions (27) and (28) to prevent possible instability in numerical simulations.[36] Since the constant 1 is small enough not to significantly affect the parameter estimation, as shown in Ref. 36, correlations (27) and (28) may be replaced by the following:

$$Nu_{ws}(t) = 0.64\xi^{1.79}Pr^{0.72}Re(t)^{0.72} \tag{29}$$

$$Nu_{mf}(t) = 0.48\xi^{2.75}Pr^{0.79}Re(t)^{0.79} \tag{30}$$

Comparing Eqs. (5) and (29) as well as Eqs. (5) and (30) yields the following results:

$$K_{ws} = 0.64\xi^{1.79}Pr^{0.72} \qquad n_{ws} = 0.72 \tag{31}$$

$$K_{mf} = 0.48\xi^{2.75}Pr^{0.79} \qquad n_{mf} = 0.79 \tag{32}$$

Therefore, $\beta_{F,\mathrm{ws}} = 1.84697$ and $\beta_{F,\mathrm{mf}} = 1.88203$.[37] To calculate the thermal capacity of the matrix per unit of volume, $c_s\rho_s$, appearing in Eq. (3), the temperature matrix has been assumed equal to the mean effective temperature of the working gas in the regenerator. The thermal properties of some metals and alloys, having application to Stirling gas circuit design, are given in Ref. 33.

A very simple expression (but quite approximate) to calculate the Stirling regenerator effectiveness is given by Benvenuto and de Monte.[30]

Machine Thermal Efficiency

The heat transfer rates at the heater $\bar{\dot{Q}}_h$ and cooler $\bar{\dot{Q}}_k$ may be calculated by using the ε − NTU method:

$$\bar{\dot{Q}}_h = \varepsilon_h C_{\mathrm{hf}}(T_{\mathrm{hf,in}} - T_{w,h}) \tag{33}$$

$$\bar{\dot{Q}}_k = \varepsilon_k C_{kf}(T_{w,k} - T_{kf,\mathrm{in}}) \tag{34}$$

However, $\bar{\dot{Q}}_h$ and $\bar{\dot{Q}}_k$ may be calculated also applying the energy balance equation to the hot and cold spaces (working fluid side), shown in Fig. 3. Applying the energy equation to the hot space

$$\dot{U}_{w,\mathrm{hot}} = \dot{Q}_h - \dot{W}_e - C_w T_{w,h/\mathrm{rg}} \tag{35}$$

where U is the internal energy and $T_{w,h/\mathrm{rg}}$ is

$$T_{w,h/\mathrm{rg}} = \begin{cases} T_{w,h} & \text{if} \quad \dot{m}_w > 0 \\ T_{w,\mathrm{ro\text{-}hi}} & \text{if} \quad \dot{m}_w < 0 \end{cases}$$

The cyclic average value of Eq. (35) gives

$$\bar{\dot{Q}}_h = \bar{\dot{W}}_e + f \oint C_w T_{w,h/\mathrm{rg}}\, \mathrm{d}t \tag{36}$$

where the integral appearing on the right side of Eq. (36) represents the average heat transfer rate supplied in excess over a cycle by the heating fluid because of the ineffectiveness of the regenerator. Therefore, Eq. (36) may be also written as follows:

$$\bar{\dot{Q}}_h = \bar{\dot{W}}_e + \hat{\dot{Q}}_{\mathrm{rg}}(1 - \varepsilon_{\mathrm{rg}}) \tag{37}$$

where $\hat{\dot{Q}}_{\mathrm{rg}} = c_{p,w}\hat{\dot{m}}_w(T_{w,h} - T_{w,k})$ is the average heat transfer rate transferred from the working gas to the regenerator matrix in case of ideal behavior of the regenerator. It may be more accurately calculated as shown in Ref. 30, and depends on the thermodynamic model (e.g., either isothermal or adiabatic) selected for the working gas circuit. In case of a perfect regenerator: $T_{w,h/\mathrm{rg}} = T_{w,h}$ at any time and, therefore, $\bar{\dot{Q}}_h = \bar{\dot{W}}_e$. Similarly, for the cold space

$$\bar{\dot{Q}}_k = \bar{\dot{W}}_c + \hat{\dot{Q}}_{\mathrm{rg}}(1 - \varepsilon_{\mathrm{rg}}) \tag{38}$$

The cyclic average powers $\bar{\dot{W}}_e$ and $\bar{\dot{W}}_c$, delivered by the expansion and compression spaces, are

$$\bar{\dot{W}}_e = \oint p_w\, \mathrm{d}V_e \qquad \bar{\dot{W}}_c = -\oint p_w\, \mathrm{d}V_c$$

Linearizing the cycle gas pressure and assuming sinusoidal the motions of the piston and displacer, the cyclic average powers are

$$\bar{\dot{W}}_e = -\frac{1}{2}\,\omega\left(\frac{X_p}{2}\right)^2 r \sin\phi A_d \left(\frac{\partial p_w}{\partial x_p}\right)_{0,0} \tag{39}$$

$$\bar{\dot{W}}_c = -\frac{1}{2}\,\omega\left(\frac{X_p}{2}\right)^2 r \sin\phi \left[(A_d - A_r)\left(\frac{\partial p_w}{\partial x_p}\right)_{0,0} + A_p\left(\frac{\partial p_w}{\partial x_d}\right)_{0,0}\right] \tag{40}$$

Comparing Eqs. (33) and (37) as well as Eqs. (34) and (38) yields two nonlinear algebraic equations in two unknown variables: $T_{w,k}$ and $T_{w,h}$

$$\varepsilon_h C_{\mathrm{hf}}(T_{\mathrm{hf,in}} - T_{w,h}) = \bar{\dot{W}}_e + \hat{\dot{Q}}_{\mathrm{rg}}(1 - \varepsilon_{\mathrm{rg}}) \tag{41}$$

$$\varepsilon_k C_{kf}(T_{w,k} - T_{kf,\mathrm{in}}) = \bar{\dot{W}}_c + \hat{\dot{Q}}_{\mathrm{rg}}(1 - \varepsilon_{\mathrm{rg}}) \tag{42}$$

Once the temperatures $T_{w,k}$ and $T_{w,h}$ are calculated, it is possible to estimate readily the heater, cooler, and regenerator effectiveness, and their effect on thermal efficiency of the machine, defined as

$$\eta = (\bar{\dot{Q}}_h - \bar{\dot{Q}}_k)/\bar{\dot{Q}}_h = (\bar{\dot{W}}_e - \bar{\dot{W}}_c)/\bar{\dot{Q}}_h$$

An iterative procedure is necessary to solve Eqs. (41) and (42), since the thermal properties of the outside fluids (appearing in the dimensionless groups of the employed heat transfer correlations) have to be calculated at the temperature $(T_{\mathrm{hf,in}} + T_{\mathrm{hf,out}})/2$ for the heater, and temperature $(T_{kf,\mathrm{in}} + T_{kf,\mathrm{out}})/2$ for the cooler.

The outlet temperatures of the heating and cooling fluids may be evaluated applying the energy balance equations to the heater (heating fluid side) and cooler (cooling fluid side), respectively:

$$\bar{\dot{Q}}_h = C_{\mathrm{hf}}(T_{\mathrm{hf,in}} - T_{\mathrm{hf,out}})$$

$$\bar{\dot{Q}}_k = C_{kf}(T_{kf,\mathrm{out}} - T_{kf,\mathrm{in}})$$

Free-Piston Stirling Machines

In FPSEs the dynamics, the thermodynamics, the heat-exchange, and the behavior of the adopted load device are strictly coupled among them, because of the lack of mechanical linkages able to fix strokes (X_p, X_d) and phasing (ϕ) for the moving elements.

The parameters of free-piston machines, particularly suitable for space power applications because of their high efficiency and reliability,[38] are linked among them by means of the following four algebraic equations[12]:

$$\beta - \gamma/\alpha = \delta\alpha/\gamma \tag{43}$$

$$\omega^2 = \gamma/\alpha \tag{44}$$

$$\tan\phi = \frac{\omega[D'_{\mathrm{pp}}S'_{\mathrm{pd}} + D'_{\mathrm{pd}}(\omega^2 - S'_{\mathrm{pp}})]}{S'_{\mathrm{pd}}S'_{\mathrm{pp}} + \omega^2(D'_{\mathrm{pd}}D'_{\mathrm{pp}} - S'_{\mathrm{pd}})} \tag{45}$$

$$r = \frac{\Omega^2 + \omega^2 D'^2_{\mathrm{pp}}}{[(S'_{\mathrm{pd}}\Omega + \omega^2 D'_{\mathrm{pp}}D'_{\mathrm{pd}})^2 + \omega^2(D'_{\mathrm{pd}}\Omega - D'_{\mathrm{pp}}S'_{\mathrm{pd}})^2]^{1/2}} \tag{46}$$

where $\Omega = (S'_{\mathrm{pp}} - \omega^2)$, and α, β, γ, and δ depend on the stiffness S'_{ij} and damping D'_{ij} $(i, j = p, d)$ coefficients per unit of moving element mass of the machine. These coefficients are not constant, but depend on X_p, r, ω, ϕ, as shown in Ref. 12. The average useful power $\bar{\dot{W}}_u$ delivered by the machine in correspondence with the engine/load device interface is

$$\bar{\dot{W}}_u = (\tfrac{1}{2})b_{\mathrm{ld}-l,D}D_{\mathrm{pp,ld}-l}(\omega X_p/2)^2 \tag{47}$$

where the constants $b_{\mathrm{ld}-l,D}$ and $D_{\mathrm{pp,ld}-l}$ depend on the load device connected to the engine.[12,13] Another basic functional constraint that links the various machine parameters is

$$\alpha, \beta, \gamma, \delta > 0 \tag{48}$$

In particular, Eqs. (43) and (48), if verified, ensure the engine of a cyclic steady operation.[12]

Equations (43–47) represent the dynamic behavior of any FPSE system, whereas the influence of a particular load device as well as the thermodynamic behavior of a specific FPSE are described by means of the S'_{ij} and D'_{ij} coefficients, which have been defined in Refs. 12 and 13. Their algebraic expressions depend both on the adopted load device and on the thermodynamic model (e.g., either isothermal or adiabatic) selected for the working gas circuit of the machine. In particular, the isothermal $S'_{ij,w}$ and $D'_{ij,w}$ coefficients are given in Refs. 5 and 34, as well as the adiabatic $S'_{ij,w}$ and $D'_{ij,w}$ coefficients given in Ref. 35.

In both the isothermal and adiabatic models, the heater and cooler walls are maintained isothermally at the constant source and sink temperatures, and the heat exchanger cycle gas temperatures are equal to their associated wall temperatures. In addition, the regenerator effectiveness is considered unitary. Therefore, the heat exchangers (including the regenerator) are perfectly effective ($\varepsilon_h = \varepsilon_k = \varepsilon_{rg} = 1$).

The closed-form thermal analysis developed in this work allows the heat transfer processes at the FPSE heater, cooler and regenerator, described by Eqs. (41) and (42), to be coupled (although not inherently) to the FPSE dynamic (including the load device) and thermodynamic behavior, described by Eqs. (43–47). Thus, the complete FPSE model is obtained simply linking Eqs. (41) and (42) to Eqs. (43–47), and it may be used for both the performance prediction and the design of the machine. In the first case, the six unknown variables X_p, r, ω, ϕ, $T_{w,h}$, and $T_{w,k}$ may be obtained by solving Eqs. (41–46). Then, Eq. (47) allows $\bar{\dot{W}}_u$ to be calculated.

In the second case, once $\bar{\dot{W}}_u$ and ω are assigned, and some other data are fixed on the basis of experience, it is possible to obtain the remaining variables (seven) by solving Eqs. (41–47). The restriction (48) has to be verified too.

Effect of Heat-Exchange Effectiveness on SPRE Performance

To show the possible applications of the developed theory, the Space Power Research Engine has been considered. It is a 12.5-kW single-cylinder free-piston Stirling engine with displacer sprung to ground connected to a linear alternator load. The SPRE design operating conditions are[1–3]: helium as working gas; pressure of 150 bar when the engine does not work; inlet temperature and volumetric flow rate of the coolant (mixture of water and ethyl glycol) equal to 50.2°C and 1.065 l/s, respectively; inlet temperature and volumetric flow rate of the heating fluid (molten salt called HITEC) equal to 384°C and 1.5 l/s, respectively; and resistive load equal to 1.6655 Ω, that fixes the power piston stroke to 2.03 cm.

The heater contains 1632 tubes brazed at each end into holes in annular tube sheets, as shown in Fig. 2. The helium working gas is on the inside of the tubes and the molten salt heating fluid flows over the outside of the tubes. The regenerator is a simple stack of many stainless-steel wire mesh screens stamped to the diametral dimensions of the regenerator section of the heater assembly. The configuration of the cooler is very similar to the heater. It contains 1584 tubes and is basically an annular box.

Table 1, obtained assuming an adiabatic behavior for the working spaces,[5,35] shows the effect on SPRE performance of each heat-exchange effectiveness considered separately from the other ones. The parameter $\Delta\eta/\eta$ represents the percent decrease (%) of machine thermal efficiency because of nonunitary heat transfer effectiveness. In particular, Table 1 shows that in the SPRE free-piston engine a small deviation from the ideal behavior of the regenerator (1.8%) can lead to a substantial decrease of the engine efficiency (−12.2%). On the contrary, a very large deviation from the perfect behavior of the heater and cooler (46 and 25%, respectively) leads to a small decrease of the efficiency (−4.1 and −2%, respectively). Therefore, the regenerator can be considered as the heart of the Stirling machine. It substantially improves the efficiency of the cycle,[9,39] storing heat during one part of the cycle for reuse during another part, but the sensitivity of thermal efficiency to regenerator behavior is very high.

Table 1 Effect of the various heat-exchange effectiveness on SPRE thermal efficiency

ε_h	ε_{rg}	ε_k	η	$\Delta\eta/\eta$, %
1	1	1	0.49	0
0.54	1	1	0.47	−4.1
1	0.982	1	0.43	−12.2
1	1	0.75	0.48	−2
0.54	0.982	0.75	0.41	−16.3

Conclusions

A fully analytical procedure has been developed for the evaluation of the effectiveness of the heat exchangers and regenerator in Stirling cycle machines. The proposed procedure allows the following:

1) The actual operating conditions of the machine (volumetric flow rates and inlet temperatures of the outside fluids, including the load) to be considered.

2) A ready estimation of the heat transfer effects of the three heat exchangers on the efficiency of any running Stirling machine (e.g., either kinematic or free-piston) to be obtained.

3) The heat exchangers and regenerator of kinematic engines to be designed, although in a preliminary way. The design has to be considered preliminary, and therefore approximate, because of the various simplifying assumptions made in the thermal analysis in order to obtain an analytical description of the heat transfer processes.

4) The heat exchangers and regenerator of free-piston engines to be designed in a proper way, although approximate. The inherent approximation is necessary since the design of an FPSE requires an analytical description, as already stated. The flow losses through the heat exchangers and regenerator are considered by means of the D'_{ij} damping coefficients,[12] appearing in the FPSE basic equations: Eqs. (41–47). The FPSE heater and cooler are optimized in a fashion that looks at heat transfer, flow dissipation, and volumetric compression ratio of the machine and its stability. It will be noted that the optimum tends to operate in a turbulent flow regime. Turbulent flow heat exchangers can compromise the performance of the machine, but improve considerably its stability,[13,14] which is very important when large variations of the operational parameters are required.

5) The response of the FPSE to changes of the actual operating conditions to be studied, as volumetric flow rates and inlet temperatures of the outside fluids.

Therefore, the described methodology is particularly suitable for FPSEs, characterized by sinusoidal motions of the moving elements. Although it has been developed by the author only for beta machines, it can be applied easily to any Stirling arrangement (alpha and gamma types).

References

[1] Brown, A. T., "Space Power Demonstrator Engine, Phase I Final Report," NASA CR-179555, May 1987.

[2] Goldberg, L. F., "One- and Two-Dimensional Stirling Machine Simulation Using Experimentally Generated Reversing Flow Turbulence Models," NASA CR-185285, Aug. 1990.

[3] Geng, S. M., and Tew, R. C., "Comparison of GLIMPS and HFAST Stirling Engine Code Predictions with Experimental Data," *Proceedings of the 27th Intersociety Energy Conversion Engineering Conference* (San Diego, CA), Society of Automotive Engineers, Warrendale, PA, 1992, pp. 5.53–5.58.

[4] Benvenuto, G., de Monte, F., and Galli, G., "The Influence of Thermodynamic Losses on Free-Piston Stirling Engine Performance," *Proceedings of the 30th Intersociety Energy Conversion Engineering Conference* (Orlando, FL), Vol. 3, American Society of Mechanical Engineers, New York, 1995, pp. 443–450.

[5]Urieli, I., and Berchowitz, D. M., "Stirling Cycle Engine Analysis," Adam Higler Ltd., Bristol, England, UK, 1984.

[6]Chen, N. C. J., Griffin, F. P., and West, C. D., "Linear Harmonic Analysis for Stirling Machines and Second Law Analysis of Four Important Losses," *Proceedings of the 19th Intersociety Energy Conversion Engineering Conference* (San Francisco, CA), American National Standards Inst., New York, 1984, pp. 1973–1981.

[7]Huang, S. C., "HFAST—A Harmonic Analysis Program for Stirling Cycles," *Proceedings of the 27th Intersociety Energy Conversion Engineering Conference* (San Diego, CA), Vol. 5, Society of Automotive Engineers, Warrendale, PA, 1992, pp. 5.47–5.52.

[8]Bauwens, L., "Semi-Analytical Stirling Engine Models: Stratified Isentropic Cylinders and Quasi-Steady Heat Exchangers," *Proceedings of the 29th Intersociety Energy Conversion Engineering Conference* (Monterey, CA), AIAA, Washington, DC, 1994, pp. 1841–1846.

[9]Reader, G. T., "The Pseudo Stirling Cycle—A Suitable Performance Criterion?" *Proceedings of the 13th Intersociety Energy Conversion Engineering Conference* (San Diego, CA), Society of Automotive Engineers, Warrendale, PA, 1978, pp. 1763–1770.

[10]Kankam, M. D., and Rauch, J. S., "Comparative Survey of Dynamic Analyses of Free-Piston Stirling Engines," *Proceedings of the 26th Intersociety Energy Conversion Engineering Conference* (Boston, MA), Vol. 5, American National Standards Inst., New York, 1991, pp. 314–319.

[11]Kankam, M. D., and Rauch, J. S., "Transient and Steady-State Performance of Space Power Research Engine with Resistive/Motor Loads," *Proceedings of the 31st Intersociety Energy Conversion Engineering Conference* (Washington, DC), Vol. 2, Inst. of Electrical and Electronics Engineers, New York, 1996, pp. 1277–1282.

[12]Benvenuto, G., and de Monte, F., "Analysis of Free-Piston Stirling Engine/Linear Alternator Systems. Part 1: Theory," *Journal of Propulsion and Power*, Vol. 11, No. 5, 1995, pp. 1036–1046.

[13]Benvenuto, G., and de Monte, F., "Analysis of Free-Piston Stirling Engine/Linear Alternator Systems. Part 2: Results," *Journal of Propulsion and Power*, Vol. 11, No. 5, 1995, pp. 1047–1055.

[14]Benvenuto, G., and de Monte, F., "The Effect of Nonlinear Thermo-Fluid-Dynamic Terms on Free-Piston Stirling Machine Stability," *Proceedings of the 31st Intersociety Energy Conversion Engineering Conference* (Washington, DC), Vol. 2, Inst. of Electrical and Electronics Engineers, New York, 1996, pp. 1224–1231.

[15]Kays, W. M., and London, A., *Compact Heat Exchangers*, 2nd ed., McGraw–Hill, New York, 1964.

[16]Ozisik, M. N., *Heat Transfer, A Basic Approach*, McGraw–Hill, Singapore, 1985.

[17]Jakob, M., *Heat Transfer*, Vol. II, Wiley, New York, 1957.

[18]Baclic, B. S., and Heggs, P. J., "Unified Regenerator Theory and Reexamination of the Unidirectional Regenerator Performance," *Advances in Heat Transfer*, Vol. 20, Academic, San Diego, CA, 1990, pp. 133–179.

[19]Nusselt, W., "Die Theorie des Winderhitzers," *Zeitscrift des Vereins Deutsches Ingenieure*, Vol. 71, 1927.

[20]Rea, S. N., and Smith, J. L., Jr., "The Influence of Pressure Cycling on Thermal Regenerators," *Journal of Engineering for Industry*, Vol. 89, No. 3, Series B, Aug. 1967, pp. 563–569.

[21]Qvale, E. B., and Smith, J. L., Jr., "An Approximate Solution for the Thermal Performance of a Stirling-Engine Regenerator," *Journal of Engineering for Power*, Vol. 91, April 1969, pp. 109–112.

[22]Jones, J. D., "Performance of a Stirling Engine Regenerator Having Finite Mass," *Journal of Engineering for Gas Turbines and Power*, Vol. 108, Oct. 1986, pp. 669–673.

[23]Jones, J. D., "Optimization of Stirling Engine Regenerator Design," *Proceedings of the 25th Intersociety Energy Conversion Engineering Conference* (Reno, NV), Vol. 5, AIChE, 1990, pp. 359–365.

[24]Organ, A. J., "The Wire Mesh Regenerator of the Stirling Cycle Machine," *International Journal of Heat and Mass Transfer*, Vol. 37, No. 16, 1994, pp. 2525–2534.

[25]Organ, A. J., "Solution of the Classic Thermal Regenerator Problem," *Journal of Mechanical Engineering Science*, Vol. 208, No. C3, 1994, pp. 187–197.

[26]Hargreaves, C. M., *The Philips Stirling Engine*, Elsevier, Amsterdam, The Netherlands, 1991.

[27]Chung, W., and Kim, S., "Heat Transfer Analysis in Stirling Engine Heat Input System," *Proceedings of the 30th Intersociety Energy Conversion Engineering Conference* (Orlando, FL), Vol. 3, American Society of Mechanical Engineers, New York, 1995, pp. 391–396.

[28]Dochat, G., "Free-Piston Space Stirling Technology Program. An Update," *Proceedings of the 25th Intersociety Energy Conversion Engineering Conference* (Reno, NV), Vol. 5, AIChE, 1990, pp. 219–223.

[29]Walker, G., and Senft, J. R., *Free Piston Stirling Engines*, Springer–Verlag, Berlin, 1985.

[30]Benvenuto, G., and de Monte, F., "A Simple Approach to Calculate the Regenerator Effectiveness in Stirling Machines," *Proceedings of the 7th International Conference on Stirling Cycle Machines* (Tokyo), JSME, 1995, pp. 307–313.

[31]Tang, X., and Cheng, P., "Correlations of the Cycle-Averaged Nusselt in a Periodically Reversing Pipe Flow," *International Communications in Heat and Mass Transfer*, Vol. 20, 1993, pp. 161–172.

[32]Simon, T. W., and Seume, J. R., "A Survey of Oscillating Flow in Stirling Engine Heat Exchangers," NASA CR-182108, March 1988.

[33]Organ, A. J., *Thermodynamics and Gas Dynamics of the Stirling Cycle Machine*, Cambridge Univ. Press, Cambridge, England, UK, 1992.

[34]Benvenuto, G., de Monte, F., and Farina, F., "Dynamic Behavior Prediction of Free-Piston Stirling Engines," *Proceedings of the 25th Intersociety Energy Conversion Engineering Conference*, Vol. 5, AIChE, 1990, pp. 346–351.

[35]Benvenuto, G., and de Monte, F., "An Adiabatic Approach for the Design and the Response-to-Load-Evaluation of Free-Piston Stirling Engines," *Proceedings of the 27th Intersociety Energy Conversion Engineering Conference* (San Diego, CA), Society of Automotive Engineers, Warrendale, PA, 1992, pp. 5.23–5.32.

[36]Gedeon, D., and Wood, J. G., "Oscillating-Flow Regenerator Test Rig: Hardware and Theory with Derived Correlations for Screens and Felts," NASA CR-198442, Feb. 1996.

[37]Abramowitz, M., and Stegun, I. A., *Handbook of Mathematical Functions*, Dover, New York, 1972.

[38]Underwood, M. L., "Advanced Radioisotope Power Source Options for Pluto Express," *Proceedings of the 30th Intersociety Energy Conversion Engineering Conference* (Orlando, FL), Vol. 1, American Society of Mechanical Engineers, New York, 1995, pp. 625–630.

[39]Walker, G., *Stirling Engines*, Oxford Univ. Press, Oxford, England, UK, 1980.